Applied Mathematical Sciences, Vol. 11, 2017, no. 54, 2663 - 2670
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ams.2017.78252

A New Method to Solve Assignment Models

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Abstract

Assignment models is one of topics of operations research. It consists of assigning a specific (person or worker) to a specific (task or job) assuming that there are the number of persons equal to the number of tasks available. The optimal result is to assignment one person to one job, contrast to the transportation models the source is connected to one or more of destination. The most common method to solve assignment models is the Hungarian method. In this paper introduced another method to solve assignment models by use the graph in the general formula directly.
The edges are represented the cost of assigning person to task, the nods are represented the tasks and persons. The solution will be by choosing the minimum cost (edge) from the costs (edges) and delete the selected edge as well as nodes associated with the edge, then delete all other edges associated with the nodes. Repeat the process until all workers are assigned to each tasks and be the solution is the optimal solution.

Keywords: Assignment problems, Hungarian method, Graph theory

1 Introduction

"The best person for the job" is an apt description of the assignment model. [3] An important topic, put forward immediately after the transportation problem, is the assignment problem. This is particularly important in the theory of decision making. [4] The assignment models is a special state of a linear programming models and its similar to the transportation model. The different between the assignment models and transportation models is the focuses on the fact that transportation models assignment the multiple sources to multiple destinations while the assignment models assignment one source to one destinations, which is a special case of transportation models. [7]

The situation can be illustrated by the assignment of workers with varying degrees of skill to jobs. [5]
The assignment models arises because available resources such as (workers, jobs or machines etc.) have varying degrees of skills for showing the different activities. [1]
Therefore, distance, time, profit or cost of performing different activities are be different. The assignment models are completely specified by its two components, the assignment which represents the underlying combinatorial structure and the objective function to be optimized. [1]
Assignment models deals with the topic how to assign n workers to n jobs such that the cost incurred is minimized. [7]
It was developed and published in 1955 by H. Kuhn, who gave the name "Hungarian method" because the method in general based on the earlier works of two Hungarian mathematicians: D. Kőnig and J. Egerváry and is therefore known as Hungarian method of assignment models. [1], [2]
Each assignment models has a matrix or table connected with it. Generally the row include the peoples or objects we wish to assign, and the column include the tasks or jobs we want them assigned. Consider a problem of assignment of n sources (resource) to n destinations (activities) so as to minimum the overall time or cost in such a way that each sources can connect with one and only one destination. The general Graph theory of assignment model is given as under. [4], [7]

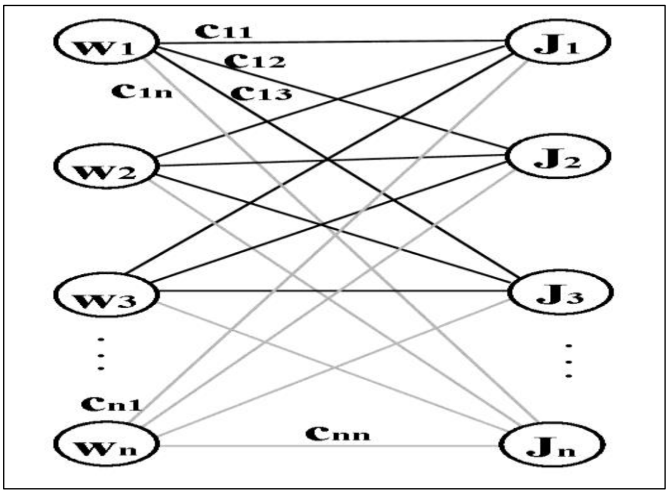


Figure (1): Network representation of the assignment problem

The cost matrix (Cij) is given as under. [7]

Workers (persons)	Jobs (Tasks) J_1	J_2	J_3	……	J_n
W_1	C_{11}	C_{12}	C_{13}	……	C_{1n}
W_2	C_{21}	C_{22}	C_{23}	……	C_{2n}
W_3	C_{31}	C_{32}	C_{33}	……	C_{3n}
:	:	:	:		:
W_n	C_{n1}	C_{n2}	C_{n3}	……	C_{nn}

Figure (2): Matrix of Costs

Let the cost of i^{th} persons assigned to j^{th} jobs be represent by c_{ij}.
Let the number of unites by assignment the persons i^{th} to a jobs j^{th} be represent by X_{ij}.

$X_{ij} = 1$ if **i** person is assigned **j** job.

Or $X_{ij} = 0$ if **i** person is not assigned **j** job.

The objective function is:

$$\text{Min } Z= \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to the constraint

$$\sum_{i=1}^{n} X_{ij} = 1 \quad \text{and} \quad \sum_{j=1}^{n} X_{ij} = 1, \quad X_{ij} = 1 \text{ or } 0$$

For all i=1,2,...,n and j =1,2,...,n [3], [5], [7]

2 Hungarian Method

Step (1): select the smallest number from each row and subtract it from the other numbers in the same row as well as for the columns.
Step (2): Draw the minimum number of vertical and horizontal lines to cover all zeros in all rows and columns in the matrix. Let the minimum number of lines is L and the number of columns or rows is n.
If L = n, the matrix then an optimal assignment can be find, then proceed to step (5).
If L < n then proceed to step (3).
Step (3): Determine the smallest number in the matrix, not covered by L lines. Subtract this minimum number from all uncovered numbers and add the same number at the intersection of horizontal and vertical lines.
Step (4): Repeat step (2) and step (3) until L = n become.

Step (5): To find assignment by test assigning all the zeroes in the rows and columns. The solution is optimal when assigning one and only one zero per row and column in given matrix.
Step (6): Repeat the step (5) until to exactly find one zero to be assignment in each row (column), then process ends.
Step (7): Write the numbers that corresponding to the zeros assigned in the previous step in the main matrix and calculate the objective function. [7]

3 The proposed method

In the proposed model use the graph theory depends of the assignment model and according to the following steps:
1. Converting the problem into a graph theory according to the general formulation of the assignment model.
2 - Choose the lowest cost between workers and tasks.
3. Delete the selected edge as well as the nods associated with edge.
4. Repeat the previous step to obtain each worker associated with only one task.
5. Calculate the objective function.
6. When exist more than one edges has the same cost, we find a multi-solution and find the objective function for each solution and choose the lowest value for the objective function.

4 Example 1

Four jobs J1, J2, J3 and J4 are to be assigned to four persons P1, P2, P3 and P4. The processing costs are given in the following matrix. Find the optimal assignment which will lower the total processing cost.

		jobs			
		J_1	J_2	J_3	J_4
persons	P_1	8	20	15	17
	P_2	15	16	12	10
	P_3	22	19	16	30
	P_4	25	15	12	9

First: Solution by Hungarian method.
Step (1): Row subtraction

		jobs			
		J_1	J_2	J_3	J_4
persons	P_1	0	12	7	9
	P_2	5	6	2	0
	P_3	6	3	0	14
	P_4	16	6	3	0

Step (2): Column subtraction

		jobs			
		J_1	J_2	J_3	J_4
persons	P_1	0	9	7	9
	P_2	5	3	2	0
	P_3	6	0	0	14
	P_4	16	3	3	0

Step (3):

		jobs			
		J_1	J_2	J_3	J_4
persons	P_1	0	9	7	9
	P_2	5	3	2	0
	P_3	6	0	0	14
	P_4	16	3	3	0

Step (4):

		jobs			
		J_1	J_2	J_3	J_4
persons	P_1	0	7	5	9
	P_2	5	1	0	0
	P_3	8	0	0	16
	P_4	16	1	1	0

		jobs			
		J_1	J_2	J_3	J_4
persons	P_1	0	7	5	9
	P_2	5	1	0	0
	P_3	8	0	0	16
	P_4	16	1	1	0

Step (5) and Step (6):

		jobs			
		J_1	J_2	J_3	J_4
persons	P_1	$\boxed{0}$	7	5	9
	P_2	5	1	$\boxed{0}$	$\cancel{0}$
	P_3	8	$\boxed{0}$	$\cancel{0}$	16
	P_4	16	1	1	$\boxed{0}$

Step (7):

So the optimal assignment for all persons to jobs as shown in the following:

$P_1 \longrightarrow J_1 = 8$

$P_2 \longrightarrow J_3 = 12$

$P_3 \longrightarrow J_2 = 19$

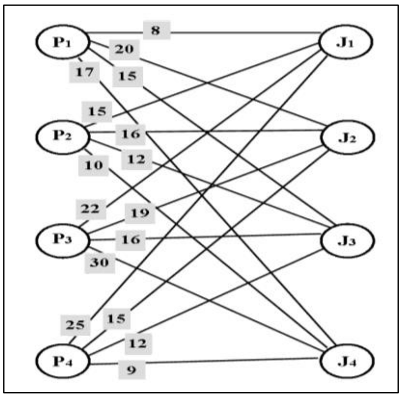
$P_4 \longrightarrow J_4 = 9$

Total minimum cost is $Z = \sum_{i=1}^{4}\sum_{j=1}^{4} c_{ij} x_{ij}$

$Z = 8 \times 1 + 12 \times 1 + 19 \times 1 + 9 \times 1 = 48$

Second: Solution by proposed method

Step (1):

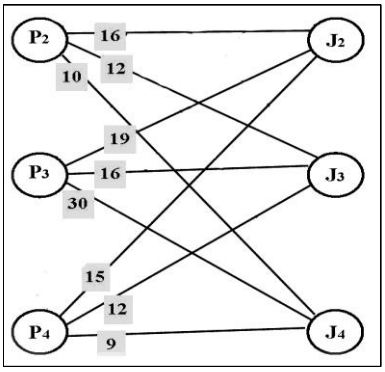


Step (2):

Min cost $P_1 \rightarrow J_1 = 8$

Step (3):

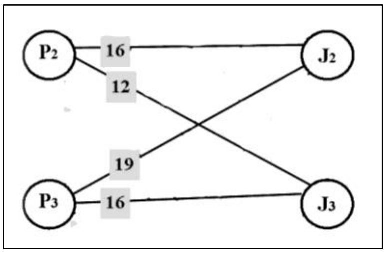
Delete P_1 and J_1



Step (4):

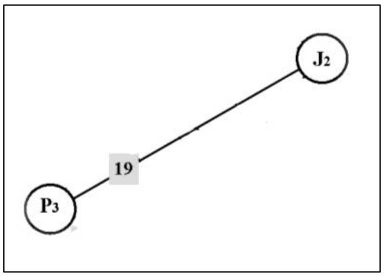
Min cost $P_4 \rightarrow J_4 = 9$

Delete P_4 and J_4



Min cost $P_2 \rightarrow J_3 = 12$

Delete P_2 and J_3



Min cost $P_3 \rightarrow J_2 = 19$

Step (5):

Total minimum cost is Z= $\sum_{i=1}^{4}\sum_{j=1}^{4} c_{ij}x_{ij}$

Z= 8 × 1+ 9 × 1+ 12× 1+9× 1 = 48

5 Conclusion

The proposed method is easier and faster than the known Hungarian method and the final output of the proposed method is similar to an solution to produced when using the Hungarian method.

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Received: August 24, 2017; Published: October 19, 2017